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BF Models in Dual Formulations of Linearized Gravity

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The case of couplings in $D = 5$ between a simple, maximal BF model and the dual formulation of linearized gravity is considered. All the possible interactions are exhausted by means of computing the "free" local BRST cohomology in ghost number zero.

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The power of the BRST formalism was clearly proved by the reformulation of the construction of consistent interactions in gauge theories [1, 2] in terms of the deformation theory [3–5], or, actually, in terms of the deformation of the solution to the master equation. In this paper we consider the problem of constructing all consistent couplings in $D = 5$ between an Abelian BF theory with a maximal field spectrum and a massless tensor field with the mixed symmetry (2,1). The method to be used is the deformation technique of the solution to the classical master equation [3] combined with the local BRST cohomology [4, 5]. The requirements imposed on the interacting theory are: spacetime locality. analyticity of the deformations in the coupling constant, Lorentz covariance, Poincaré invariance (we do not allow explicit dependence on the spacetime coordinates), preservation of the number of derivatives on each field (the differential order of the deformed field equations is preserved with respect to the free model) and the condition that the interacting Lagrangian contains at most two space-time derivatives (like the free one).

Our starting point is a free action that describes an Abelian BF model and a massless tensor field with the mixed symmetry (2,1) in $D = 5$

$$S_0^{\mathrm{L}}[\varphi, H, A, B, \phi, K, t] = \int d^5x \left[H_\mu \partial^\mu \varphi + \frac{1}{2} B^{\mu\nu} \partial_{[\mu} A_{\nu]} + \frac{1}{3} K^{\mu\nu\rho} \partial_{[\mu} \phi_{\nu\rho]} - \frac{1}{12} \left(F_{\mu\nu\rho|\alpha} F^{\mu\nu\rho|\alpha} - 3 F_{\mu\nu} F^{\mu\nu} \right) \right] \tag{1}$$

where we employed the notations

$$F_{\mu\nu\rho|\alpha} = \partial_{[\mu} t_{\nu\rho]|\alpha}, \;\; F_{\mu\nu} = \sigma^{\rho\alpha} F_{\mu\nu\rho|\alpha}. \tag{2}$$

Everywhere in this paper we use the Minkowski metric $\sigma_{\mu\nu} = \sigma^{\mu\nu} = \mathrm{diag}(-,+,+,+,+)$ and also the 5-dimensional Levi-Civita tensor $\varepsilon^{\mu\nu\rho\lambda\sigma}$ valued like $\varepsilon^{01234} = -\varepsilon_{01234} = -1$.

The free theory possesses the generating set of gauge transformations

$$\delta_{\varepsilon,\xi} \varphi = 0, \;\; \delta_\varepsilon H^\mu = 2 \partial_\nu \varepsilon^{\mu\nu}, \;\; \delta_\varepsilon A^\mu = \partial^\mu \varepsilon, \tag{3}$$

$$\delta_\varepsilon B^{\mu\nu} = -3 \partial_\rho \varepsilon^{\mu\nu\rho}, \;\; \delta_\xi \phi_{\mu\nu} = \partial_{[\mu} \xi_{\nu]}, \;\; \delta_\xi K^{\mu\nu\rho} = 4 \partial_\lambda \xi^{\mu\nu\rho\lambda}, \tag{4}$$

$$\delta_{\theta,\chi} t_{\mu\nu|\alpha} = \partial_{[\mu} \theta_{\nu]\alpha} + \partial_{[\mu} \chi_{\nu]\alpha} - 2 \partial_\alpha \chi_{\mu\nu}, \tag{5}$$

where the gauge parameters ε, $\varepsilon^{\mu\nu}$, $\varepsilon^{\mu\nu\rho}$, ξ_μ, $\xi^{\mu\nu\rho\lambda}$, $\theta_{\mu\nu}$ and $\chi_{\mu\nu}$ are bosonic, with $\varepsilon^{\mu\nu}$, $\varepsilon^{\mu\nu\rho}$, $\xi^{\mu\nu\rho\lambda}$ and $\chi_{\mu\nu}$ completely antisymmetric and $\theta_{\mu\nu}$ symmetric. This generating set of gauge transformations is off-shell, third-order reducible and the gauge algebra is Abelian.

In order to construct the BRST symmetry of this free model, we introduce the field, ghost and antifield spectra

$$\Phi^{\alpha_0} = \left(A^\mu, H^\mu, \varphi, B^{\mu\nu}, K^{\mu\nu\rho}, \phi_{\mu\nu}, t_{\mu\nu|\alpha} \right), \tag{6}$$

$$\Phi^*_{\alpha_0} = \left(A^*_\mu, H^*_\mu, \varphi^*, B^*_{\mu\nu}, K^*_{\mu\nu\rho}, \phi^{*\mu\nu}, t^{*\mu\nu|\alpha} \right), \tag{7}$$

$$\eta^{\alpha_1} = \left(\eta, C^{\mu\nu}, \eta^{\mu\nu\rho}, G^{\mu\nu\rho\lambda}, C_\mu, S_{\mu\nu}, A_{\mu\nu} \right), \tag{8}$$

$$\eta^*_{\alpha_1} = \left(\eta^*, C^*_{\mu\nu}, \eta^*_{\mu\nu\rho}, G^*_{\mu\nu\rho\lambda}, C^{*\mu}, S^{*\mu\nu}, A^{*\mu\nu}\right), \tag{9}$$

$$\eta^{\alpha_2} = \left(C^{\mu\nu\rho}, \eta^{\mu\nu\rho\lambda}, G^{\mu\nu\rho\lambda\sigma}, C, S_\mu\right), \tag{10}$$

$$\eta^*_{\alpha_2} = \left(C^*_{\mu\nu\rho}, \eta^*_{\mu\nu\rho\lambda}, G^*_{\mu\nu\rho\lambda\sigma}, C^*, S^{*\mu}\right), \tag{11}$$

$$\eta^{\alpha_3} = \left(C^{\mu\nu\rho\lambda}, \eta^{\mu\nu\rho\lambda\sigma}\right), \;\; \eta^*_{\alpha_3} = \left(C^*_{\mu\nu\rho\lambda}, \eta^*_{\mu\nu\rho\lambda\sigma}\right), \tag{12}$$

$$\eta^{\alpha_4} = \left(C^{\mu\nu\rho\lambda\sigma}\right), \;\; \eta^*_{\alpha_4} = \left(C^*_{\mu\nu\rho\lambda\sigma}\right). \tag{13}$$

Since both the gauge generators and the reducibility functions for this model are field-independent, it follows that the BRST differential s reduces to $s = \delta + \gamma$ (where δ is the Koszul-Tate differential and γ stands for the exterior derivative along the gauge orbits).

The action of the antifield-BRST differential s can always be realized in an anticanonical form $s\cdot = (\cdot, S)$, where $(,)$ is the anticanonical structure, named antibracket. and S stands for its generator. The nilpotency of s becomes equivalent to the master equation $(S, S) = 0$. For the free model under study, S reads as

$$\begin{aligned} S = S_0^{\mathrm{L}} + \int d^5x \,\big[& A^*_\mu \partial^\mu \eta + 2H^*_\mu \partial_\nu C^{\mu\nu} - 3B^*_{\mu\nu}\partial_\rho \eta^{\mu\nu\rho} + \phi^{*\mu\nu}\partial_{[\mu}C_{\nu]} \\ &+ 4K^*_{\mu\nu\rho}\partial_\lambda G^{\mu\nu\rho\lambda} + t^{*\mu\nu|\alpha}\left(\partial_{[\mu}S_{\nu]\alpha} + \partial_{[\mu}A_{\nu]\alpha} - 2\partial_\alpha A_{\mu\nu}\right) \\ &- 3C^*_{\mu\nu}\partial_\rho C^{\mu\nu\rho} + 4\eta^*_{\mu\nu\rho}\partial_\lambda \eta^{\mu\nu\rho\lambda} - 5G^*_{\mu\nu\rho\lambda}\partial_\sigma G^{\mu\nu\rho\lambda\sigma} + C^{*\mu}\partial_\mu C \\ &+ 3S^{*\mu\nu}\partial_{(\mu}S_{\nu)} + A^{*\mu\nu}\partial_{[\mu}S_{\nu]} + 4C^*_{\mu\nu\rho}\partial_\lambda C^{\mu\nu\rho\lambda} \\ &-5\left(\eta^*_{\mu\nu\rho\lambda}\partial_\sigma \eta^{\mu\nu\rho\lambda\sigma} + C^*_{\mu\nu\rho\lambda}\partial_\sigma C^{\mu\nu\rho\lambda\sigma}\right)\big]. \end{aligned} \tag{14}$$

It has been shown in Ref. [3] that if an interacting theory can be consistently constructed, then we can associate with Eq. 14 a deformed solution

$$\begin{aligned} S \to \bar{S} &= S + \lambda S_1 + \lambda^2 S_2 + \lambda^3 S_3 \cdots \\ &= S + \lambda \int d^5x \, a + \lambda^2 \int d^5x \, b + \lambda^3 \int d^5x \, c + \cdots \end{aligned} \tag{15}$$

which is the BRST generator of the interacting theory (in the above A is known as the coupling constant or deformation parameter)

$$(\bar{S}, \bar{S}) = 0, \tag{16}$$

Projecting Eq. 16 on the various powers in the coupling constant, we find that the components of $\bar{S}$ are restricted to satisfy the equivalent tower of equations

$$(S, S) = 0 \tag{17}$$

$$2\,(S_1, S) = 0, \tag{18}$$

$$2\,(S_2, S) + (S_1, S_1) = 0, \tag{19}$$

$$(S_3, S) + (S_1, S_2) = 0, \tag{20}$$

$$\vdots$$

In view of this, the construction of consistent interactions becomes equivalent to solving Eqs. 18-20, etc. (Eq. 17 is satisfied by hypothesis, since S given by Eq. 14 is the solution of the master equation for the starting free theory).

The finding of solutions to the deformation equations relies on the computation of the local BRST cohomology of the starting free theory in ghost number zero (the ghost number is the overall degree that grades the BRST complex). According to the decomposition given by Eq. 15, S will be called k-order deformation (of the solution to the master equation). If we make the notation $S_1 = \int d^5xa$, then Eq. 18 takes the local form $sa = \partial_\mu m^\mu$, for a local current. The non-integrated density of the first-order deformation a can be naturally decomposed as a sum of three components

$$a = a^{\mathrm{BF}} + a^{\mathrm{t-t}} + a^{\mathrm{int}}, \tag{21}$$

where a^{BF} depends only on the BRST generators from the BF sector, $a^{\mathrm{t-t}}$ describes the self-interactions of the tensor field $t_{\lambda\mu|\alpha}$ and a^{int} effectively mixes both sectors. In view of these, each term from the right-hand side of Eq. 21 independently satisfies an equation of the type $sa = \partial_\mu m^\mu$. The component a^{BF} has been analyzed in detail in Ref. [6], where it has been shown to be parameterized in terms of seven arbitrary smooth functions $\{(W_a)_{a=\overline{1,6}}, \overline{M}\}$ depending only on the undifferentiated scalar field φ. Concerning the piece $a^{\mathrm{t-t}}$, it has been shown in Ref. [7] that it is completely trivial, $a^{\mathrm{t-t}} = 0$.

The general expression of the cross-coupling component a^{int} is given by the next theorem.

Theorem 1. *Under the assumptions made in the beginning of the paper, the general nontrivial form of a^{int} from decomposition 21 reads as*

$$\begin{aligned}
a^{\mathrm{int}} &= (M_1)^{\mu\nu\rho}\,\tilde{D}_{\mu\nu}S_\rho + (M_1)^{\mu\nu}\left[\left(\tilde{D}_{\alpha\mu}C_{\beta\nu} - \frac{1}{2}\tilde{F}_{\mu\nu|\alpha}S_\beta\right)\sigma^{\alpha\beta} + \frac{1}{6}\tilde{D}_{\mu\nu}S\right] \\
&\quad S^*\left(k_1C + k_2\tilde{G}\right) - 2t^*_\mu\left(k_1C^\mu + \frac{k_2}{5}\tilde{G}^\mu\right) \\
&\quad - \frac{1}{6}(M_1)^\rho\left(\tilde{D}^{\mu\nu}t_{\mu\nu|\rho} - 3\tilde{F}_{\rho\mu|\nu}C^{\mu\nu} + 2\tilde{D}_{\rho\lambda}t^\lambda\right) - \left(2k_1K^{*\mu\nu\rho} + \frac{k_2}{30}\tilde{\phi}^{*\mu\nu\rho}\right)D_{\mu\nu\rho} \\
&\quad + \varepsilon^{\mu\nu\rho\lambda\sigma}M_2B^*_{\mu\nu}D_{\rho\lambda\sigma} + \left(k_1\phi^{\mu\nu} - \frac{k_2}{20}\tilde{K}^{\mu\nu}\right)F_{\mu\nu} + \frac{1}{12}M_1\tilde{F}^{\mu\nu|\rho}t_{\mu\nu|\rho},
\end{aligned} \tag{22}$$

where k_1, k_2 are some real numbers and M_1 is an arbitrary smooth function that depends at most on the undifferentiated scalar field . In addition, any object bearing the overscript "$\sim$" is the Hodge dual of the corresponding tildeless quantity

In the above we employed the notations

$$(M_1)^{\mu\nu\rho} = \frac{dM_1}{d\varphi}C^{*\mu\nu\rho} + \frac{d^2M_1}{d\varphi^2}H^{*[\mu}C^{*\nu\rho]} + \frac{d^3M_1}{d\varphi^3}H^{*\mu}H^{*\nu}H^{*\rho} \tag{23}$$

$$(M_1)^{\mu\nu} = \frac{dM_1}{d\varphi}C^{*\mu\nu} + \frac{d^2M_1}{d\varphi^2}H^{*\mu}H^{*\nu} \tag{24}$$

$$(M_1)^\mu = \frac{dW}{d\varphi}H^{*\mu} \tag{25}$$

and $D_{\mu\nu\rho} = \partial_{[\mu}A_{\nu\rho]}$. It is now clear that the first-order deformation of the solution to the master equation reduces to $S_1 = \int d^5x(a^{\mathrm{BF}} + a^{\mathrm{int}})$ and is parameterized by eight arbitrary smooth functions $\{(W_a)_{a=\overline{1,6}}, \overline{M}, M_1\}$ depending only on the undifferentiated scalar field and by two real constants k_1 and k_2.

The next equation, responsible for the second-order deformation $S_2 = \int d^5xb$, is precisely Eq. 19, whose local form reads as $2sb + \Delta = \partial_\mu n^\mu$, where $(S_1, S_1) = \int d^5x\Delta$. The non-integrated density of the second-order deformation admits a decomposition similar to Eq. 21

$$b = b^{\mathrm{BF}} + b^{\mathrm{t-t}} + b^{\mathrm{int}}, \tag{26}$$

each of its pieces satisfying independently an equation of the type $2sb + \Delta = \partial_\mu n^\mu$, namely

$$2sb^{\mathrm{BF}} + \Delta^{\mathrm{BF}} = \partial^\mu n^{\mathrm{BF}}_\mu, \tag{27}$$

$$2sb^{\mathrm{t-t}} + \Delta^{\mathrm{t-t}} = \partial^\mu n^{\mathrm{t-t}}_\mu, \tag{28}$$

$$2sb^{\rm int}+\Delta^{\rm int}=\partial^{\mu}n_{\mu}^{\rm int}. \tag{29}$$

The solution to Eq. 27 has been given in Ref. [6], where it has been shown that the seven functions parameterizing the first-order deformation in the BF sector, $a^{\rm BF}$, are required to satisfy the following equations:

$$\frac{d\bar{M}\left(\varphi\right)}{d\varphi}W_{1}\left(\varphi\right)=0,\ \ W_{1}\left(\varphi\right)W_{2}\left(\varphi\right)=0, \tag{30}$$

$$W_{1}\left(\varphi\right)\frac{dW_{2}\left(\varphi\right)}{d\varphi}-3W_{2}\left(\varphi\right)W_{3}\left(\varphi\right)+6W_{5}\left(\varphi\right)W_{6}\left(\varphi\right)=0, \tag{31}$$

$$W_{2}\left(\varphi\right)W_{3}\left(\varphi\right)+W_{5}\left(\varphi\right)W_{6}\left(\varphi\right)=0, \tag{32}$$

$$W_{1}\left(\varphi\right)\frac{dW_{6}\left(\varphi\right)}{d\varphi}+3W_{3}\left(\varphi\right)W_{6}\left(\varphi\right)-6W_{2}\left(\varphi\right)W_{4}\left(\varphi\right)=0, \tag{33}$$

$$W_{1}\left(\varphi\right)W_{6}\left(\varphi\right)=0,\ \ W_{2}\left(\varphi\right)W_{4}\left(\varphi\right)+W_{3}\left(\varphi\right)W_{6}\left(\varphi\right)=0, \tag{34}$$

$$W_{2}\left(\varphi\right)W_{5}\left(\varphi\right)=0,\ \ W_{4}\left(\varphi\right)W_{6}\left(\varphi\right)=0. \tag{35}$$

which further ensure that we can safely take $b^{\rm BF}=0$. Eq. 28 has been investigated in Ref. [7] and has been to possess only the trivial solution $b^{\rm t-t}=0$.

The existence of general solutions to Eq. 29 is summarized by the next theorem.

Theorem 2.

i) The existence of $b^{\rm int}$ as solution to Eq. 29 requires that the functions/constants that parameterize the first-order deformation are subject to the equations

$$k_{1}W_{3}+\frac{k_{2}}{60}W_{5}=0,\ \ k_{1}W_{4}+\frac{k_{2}}{2\cdot 5!}W_{3}=0, \tag{36}$$

$$k_{1}W_{6}+\frac{k_{2}}{5!}W_{2}=0,\ \ W_{1}\frac{dM_{1}}{d\varphi}=0, \tag{37}$$

$$k_{1}M_{1}=0,\ \ k_{2}M_{1}=0. \tag{38}$$

ii) Provided Eqs. 30-38 are satisfied, the second-order deformation in the interacting sector reads as

$$\begin{aligned} b^{\rm int}=&-\frac{3}{2}\left(k_{1}\phi_{\mu\nu}-\frac{k_{2}}{20}\tilde{K}_{\mu\nu}\right)\left(k_{1}\phi^{\mu\nu}-\frac{k_{2}}{20}\tilde{K}^{\mu\nu}\right)-3M_{1}\left(k_{1}\tilde{G}^{*\mu}-\frac{k_{2}}{120}C^{*\mu}\right)S_{\mu}\\ &-\frac{1}{2}\left(k_{1}K^{*}_{\mu\nu\rho}+\frac{k_{2}}{60}\tilde{\phi}^{*}_{\mu\nu\rho}\right)\left(-M_{1}\tilde{A}^{\mu\nu\rho}+\varepsilon^{\mu\nu\rho\lambda\sigma}\left(M_{1}\right)_{\lambda}S_{\sigma}\right)\\ &-\frac{1}{4}\varepsilon^{\mu\nu\rho\lambda\sigma}\left(\left(M_{1}\right)_{\mu\nu}S_{\rho}-\left(M_{1}\right)_{\mu}A_{\nu\rho}\right)\left(k_{1}\phi_{\lambda\sigma}-\frac{k_{2}}{20}\tilde{K}_{\lambda\sigma}\right). \end{aligned} \tag{39}$$

In conclusion, the second-order deformation reduces to $S_{2}=\int d^{5}xb^{\rm int}$. It is important to emphasize that in both the first- and second-order deformations the functions $\{(W_{a})_{a=\overline{1,6}},\overline{M},M_{1}\}$ and the real constants k_{1} and k_{2}, are now subject to Eqs. 30-38.

It is easy to check that all the higher-order deformation equations can be taken to vanish on the solutions to Eqs. 30-38, $S_{k}=0$ for all $k>2$.

Putting together the results obtained so far, we can rite down the full solution of the classical master equation for the interacting theory

$$\bar{S}=S+\int d^{5}x\left(\lambda a+\lambda^{2}b\right), \tag{40}$$

From Eq. 40 we can identify the entire Lagrangian gauge structure of the interacting model. The antighost number zero piece from Eq. 40 is nothing but the Lagrangian action of the coupled theory

$$\begin{aligned}S^{\mathrm{L}}\left[\Phi^{\alpha_0}\right] = \int d^5x \Big\{ & \left(H_\mu \partial^\mu \varphi + \frac{1}{2} B^{\mu\nu} \partial_{[\mu} A_{\nu]} + \frac{1}{3} K^{\mu\nu\rho} \partial_{[\mu} \phi_{\nu\rho]} \right) \\ & + \lambda \Big[W_1 A_\mu H^\mu + W_2 B_{\mu\nu} \phi^{\mu\nu} - W_3 \phi_{[\mu\nu} A_{\rho]} K^{\mu\nu\rho} \\ & + \varepsilon^{\alpha\beta\gamma\delta\varepsilon} \left(9 W_4 A_\alpha \tilde{K}_{\beta\gamma} \tilde{K}_{\delta\varepsilon} + \frac{1}{4} W_5 A_\alpha \phi_{\beta\gamma} \phi_{\delta\varepsilon} + W_6 B_{\alpha\beta} K_{\gamma\delta\varepsilon} \right) + \bar{M}(\varphi) \Big] \\ & - \frac{1}{12} \left(F_{\mu\nu\rho|\alpha} F^{\mu\nu\rho|\alpha} - 3 F_{\mu\nu} F^{\mu\nu} \right) + \frac{\lambda}{72} \varepsilon^{\mu\nu\rho\lambda\sigma} \sigma^{\alpha\beta} M_1 F_{\mu\nu\rho|\alpha} t_{\lambda\sigma|\beta} \\ & + \lambda \left(k_1 \phi^{\mu\nu} - \frac{k_2}{20} \tilde{K}^{\mu\nu} \right) \left[F_{\mu\nu} + \frac{3\lambda}{2} \left(k_1 \phi_{\mu\nu} - \frac{k_2}{20} \tilde{K}_{\mu\nu} \right) \right] \Big\}, \end{aligned} \tag{41}$$

The antighost number one terms from Eq. 40 offer us the generating set of gauge transformations for the deformed model

$$\bar{\delta}_\Omega \varphi = -\lambda W_1 \varepsilon, \tag{42}$$

$$\begin{aligned}\bar{\delta}_\Omega H^\mu = & \, 2 D^{\mu\nu}_\nu \varepsilon + \lambda \left(\frac{dW_1}{d\varphi} H^\mu - 3 \frac{dW_3}{d\varphi} K^{\mu\nu\rho} \phi_{\nu\rho} \right) \varepsilon + 2\lambda \frac{dW_6}{d\varphi} B^{\mu\nu} \varepsilon_{\nu\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} \\ & - 3\lambda \frac{dW_2}{d\varphi} \phi_{\nu\rho} \epsilon^{\mu\nu\rho} + 2\lambda \left(\frac{dW_2}{d\varphi} B^{\mu\nu} - 3 \frac{dW_3}{d\varphi} K^{\mu\nu\rho} A_\rho \right) \xi_\nu + 12\lambda \frac{dW_3}{d\varphi} A_\nu \phi_{\rho\lambda} \xi^{\mu\nu\rho\lambda} \\ & + 3\lambda K^{\mu\nu\rho} \left(4 \frac{dW_4}{d\varphi} A_\nu \varepsilon_{\rho\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} - \frac{dW_6}{d\varphi} \varepsilon_{\nu\rho\alpha\beta\gamma} \varepsilon^{\alpha\beta\gamma} \right) \\ & + \lambda \varepsilon^{\mu\nu\rho\lambda\sigma} \left[\frac{1}{4} \frac{dW_4}{d\varphi} \varepsilon_{\nu\rho\alpha\beta\gamma} K^{\alpha\beta\gamma} \varepsilon_{\lambda\sigma\alpha'\beta'\gamma'} K^{\alpha'\beta'\gamma'} \varepsilon \right. \\ & \left. - \frac{dW_5}{d\varphi} \phi_{\nu\rho} \left(A_\lambda \xi_\sigma - \frac{1}{4} \phi_{\lambda\sigma} \epsilon \right) \right] - \frac{\lambda}{36} \varepsilon_{\alpha\beta\gamma\delta\varepsilon} \frac{dM_1}{d\varphi} t^{\alpha\beta|\mu} \partial^{[\gamma} \chi^{\delta\varepsilon]} \\ & + \frac{\lambda}{12} \varepsilon^{\mu\nu\rho\lambda\sigma} \left[\sigma^{\alpha\beta} \frac{dM_1}{d\varphi} F_{\rho\lambda\sigma|\alpha} \left(\tfrac{1}{3} \theta_{\nu\beta} + \chi_{\nu\beta} \right) + 3\lambda k_1 \frac{dM_1}{d\varphi} \phi_{\nu\rho} \chi_{\lambda\sigma} \right. \\ & \left. - \frac{2}{3} t_\nu \frac{dM_1}{d\varphi} \partial_{[\rho} \chi_{\lambda\sigma]} \right] + \frac{\lambda^2 k_2}{40} \frac{dM_1}{d\varphi} K^{\mu\nu\rho} \chi_{\nu\rho}, \end{aligned} \tag{43}$$

$$\bar{\delta}_\Omega A_\mu = \partial_\mu \varepsilon - 2\lambda W_2 \xi_\mu - 2\lambda \varepsilon_{\mu\nu\rho\lambda\sigma} W_6 \xi^{\nu\rho\lambda\sigma}, \tag{44}$$

$$\begin{aligned}\bar{\delta}_\Omega B^{\mu\nu} = & -3\partial_\rho \varepsilon^{\mu\nu\rho} - 2\lambda W_1 \varepsilon^{\mu\nu} + 6\lambda W_3 \left(2\phi_{\rho\lambda} \xi^{\mu\nu\rho\lambda} + K^{\mu\nu\rho} \xi_\rho \right) \\ & + \lambda \left(12 W_4 K^{\mu\nu\rho} \varepsilon_{\rho\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} - W_5 \varepsilon^{\mu\nu\rho\lambda\sigma} \phi_{\rho\lambda} \xi_\sigma \right), \end{aligned} \tag{45}$$

$$\begin{aligned}\bar{\delta}_\Omega \phi_{\mu\nu} = & \, D^{(-)}_{[\mu} \xi_{\nu]} + 3\lambda \left(W_3 \phi_{\mu\nu} \epsilon - 2 W_4 A_{[\mu} \varepsilon_{\nu]\alpha\beta\gamma\delta} \xi^{\alpha\beta\gamma\delta} \right) \\ & + 3\lambda \varepsilon_{\mu\nu\rho\lambda\sigma} \left(2 W_4 K^{\rho\lambda\sigma} \epsilon + W_6 \epsilon^{\rho\lambda\sigma} - \frac{k_2}{180} \partial^{[\rho} \chi^{\lambda\sigma]} \right) - \frac{\lambda^2 k_2}{40} M_1 \chi_{\mu\nu}, \end{aligned} \tag{46}$$

$$\begin{aligned}\bar{\delta}_\Omega K^{\mu\nu\rho} = & \, 4 D^{(+)}_\lambda \xi^{\mu\nu\rho\lambda} - 3\lambda \left(W_2 \epsilon^{\mu\nu\rho} + W_3 K^{\mu\nu\rho} \epsilon \right) \\ & - \lambda \varepsilon^{\mu\nu\rho\lambda\sigma} W_5 \left(A_\lambda \xi_\sigma - \tfrac{1}{2} \phi_{\lambda\sigma} \epsilon \right) - 2\lambda k_1 \partial^{[\mu} \chi^{\nu\rho]} + \frac{\lambda^2 k_1}{4} \varepsilon^{\mu\nu\rho\lambda\sigma} M_1 \chi_{\lambda\sigma}, \end{aligned} \tag{47}$$

$$\bar{\delta}_\Omega t_{\mu\nu|\alpha} = \partial_{[\mu} \theta_{\nu]\alpha} + \partial_{[\mu} \chi_{\nu]\alpha} - 2\partial_\alpha \chi_{\mu\nu} + \lambda k_1 \sigma_{\alpha[\mu} \xi_{\nu]} - \frac{\lambda k_2}{5!} \sigma_{\alpha[\mu} \varepsilon_{\nu]\beta\gamma\delta\varepsilon} \xi^{\beta\gamma\delta\varepsilon}, \tag{48}$$

where we used the collective notation $\Omega = \left(\varepsilon, \varepsilon^{\mu\nu}, \varepsilon^{\mu\nu\rho}, \xi_{\mu}, \xi^{\mu\nu\rho\lambda}, \theta_{\mu\nu}, \chi_{\mu\nu}\right)$ for the gauge parameters and also the covariant derivatives $D_{\nu} = \partial_{\nu} - \lambda \frac{dW_1}{d\varphi} A_{\nu}$ and $D_{\nu}^{(\pm)} = \partial_{\nu} \pm 3\lambda W_3 A_{\nu}$.

From the components of higher antighost number present in Eq. 40 we can read the remaining gauge structure of the interacting theory: the commutators among the deformed gauge transformations. Eqs. 42-48, and hence the properties of the deformt1 gauge algebra, their associated higher-order structure functions, and also the flew reducibility functions and relations together with their properties.

The main conclusion of this paper is that under the hypotheses of analyticity in the coupling constant, spacetime locality, Lorentz covariance, and Poincaré invariance of the deformations, combined with the preservation of the number of derivatives on each nontrivial cross-couplings in $D = 5$ between a massless tensor field with the mixed field, we find a deformation of the solution to the master equation that provides symmetry (2,1) and a topological BF model with a maximal field spectrum. The emerging Lagrangian action contains only interaction vertices of order one and two in the coupling constant that couple the massless tensor field with the mixed symmetry mainly to the scalar field, to the two-form, and to the three-form from the BF sector. Also, it is interesting to note the appearance of some self-interactions in the BF sector that are strictly due to the presence of the tensor field with the mixed symmetry (2,1) (they all vanish in its absence). The gauge transformations of all fields are deformed and, in addition, most of them include gauge parameters from the complementary sector. This is the first known case where the gauge transformations of the tensor field with the mixed symmetry (2,1) do change with respect to the free ones. The gauge algebra and the reducibility structure of the coupled model are strongly modified during the deformation procedure, becoming open and respectively on-shell, by contrast to the free theory, whose gauge algebra is Abelian and the reducibility relations hold off-shell. Our result is extremely important because dual formulations of linearized gravity have proved to be extremely rigid in allowing consistent interactions to themselves as well as to many matter or gauge theories. We think that this is the first time when a massless tensor field with the mixed symmetry $(k, 1)$ allows consistent interactions that fulfill all the working hypotheses precisely in the dimension $D = k + 3$ where it becomes dual to the Pauli-Fierz theory.

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