Propagation of electron-acoustic excitations in the presence of suprathermal background electrons

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1. Introduction

Electron-acoustic waves (EAWs) usually occur in a plasma, where a population of inertial *cool* electrons (T_c) oscillates against inertialess *hot* electrons (T_h). Two electrons populations are often characterized by a thermal Maxwellian distribution. However, some space and laboratory plasmas have such a suprathermal electron population described by a generalized Lorentzian or κ-distribution, whose behaviors are extremely different from a Maxwellian distribution. [1, 2, 3, 4] The κ-distribution function was argued to describe laboratory and space plasma observations more effectively than a Maxwellian function. [5, 6, 7]

2. Model Equations

We consider a plasma with three components, namely

- inertial cool electrons,
- inertialess hot electrons with a suprathermal distribution,
- uniform stationary ion background.

The cool electron-fluid is governed by normalized one-dimensional ($\gamma = 3$) equations as follows

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{\sigma}{n}\frac{\partial P}{\partial x}, \tag{2}$$

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} + 3P\frac{\partial u}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -(\beta+1) + n + \beta\left[1 - \frac{\phi}{(\kappa - \frac{3}{2})}\right]^{-\kappa+1/2}, \tag{4}$$

We define the density ratio and the temperature ratio as

$$\beta \equiv \frac{n_{h,0}}{n_{c,0}}, \qquad \sigma \equiv \frac{T_c}{T_h}. \tag{5}$$

where $n_{c,0}$, $n_{h,0}$, and $n_{i,0} = n_{c,0} + n_{h,0}$ are the unperturbed densities of cool electrons, hot electrons, and ions.

The cool density is normalized with the unperturbed cool density ($n_{c,0}$), the cool velocity (u_c) with the hot electron thermal velocity ($c_{h,s} = (k_B T_h/m_e)^{1/2}$), time with the inverse cool electron plasma frequency, ω_{pc}^{-1}, where $\omega_{pc} = (n_{c,0}e^2/\varepsilon_0 m_e)^{1/2}$, length with the characteristic length scale, $\lambda_0 = (\varepsilon_0 k_B T_h/n_{c,0}e^2)^{1/2}$, the wave potential ϕ with $k_B T_h/e$, and the thermal pressure P with $n_{c,0}k_B T_c$.

3. Linear Method

Linearizing Eqs. (1)–(4), we obtain the linear dispersion relation for the electron-acoustic waves:

$$\omega^2 = \frac{k^2}{k^2 + k_D^2} + 3\sigma k^2. \tag{6}$$

where $\sqrt{3\sigma}$ is the (normalized) cold electron thermal velocity. We note the appearance of the screening factor (Debye wavenumber) k_D in the denominator, defined by

$$k_D \equiv \frac{1}{\lambda_D} \equiv \left[\frac{\beta(\kappa - \frac{1}{2})}{\kappa - \frac{3}{2}}\right]^{1/2}. \tag{7}$$

From Eq. (6), we can see that the EAW frequency $\omega(k)$ increases with an increase in the wave number k and temperature ratio σ, i.e., growing $\sigma = T_c/T_h$ increases the phase speed. The density ratio (β) also manifests its physical effect in the k–ω plane.

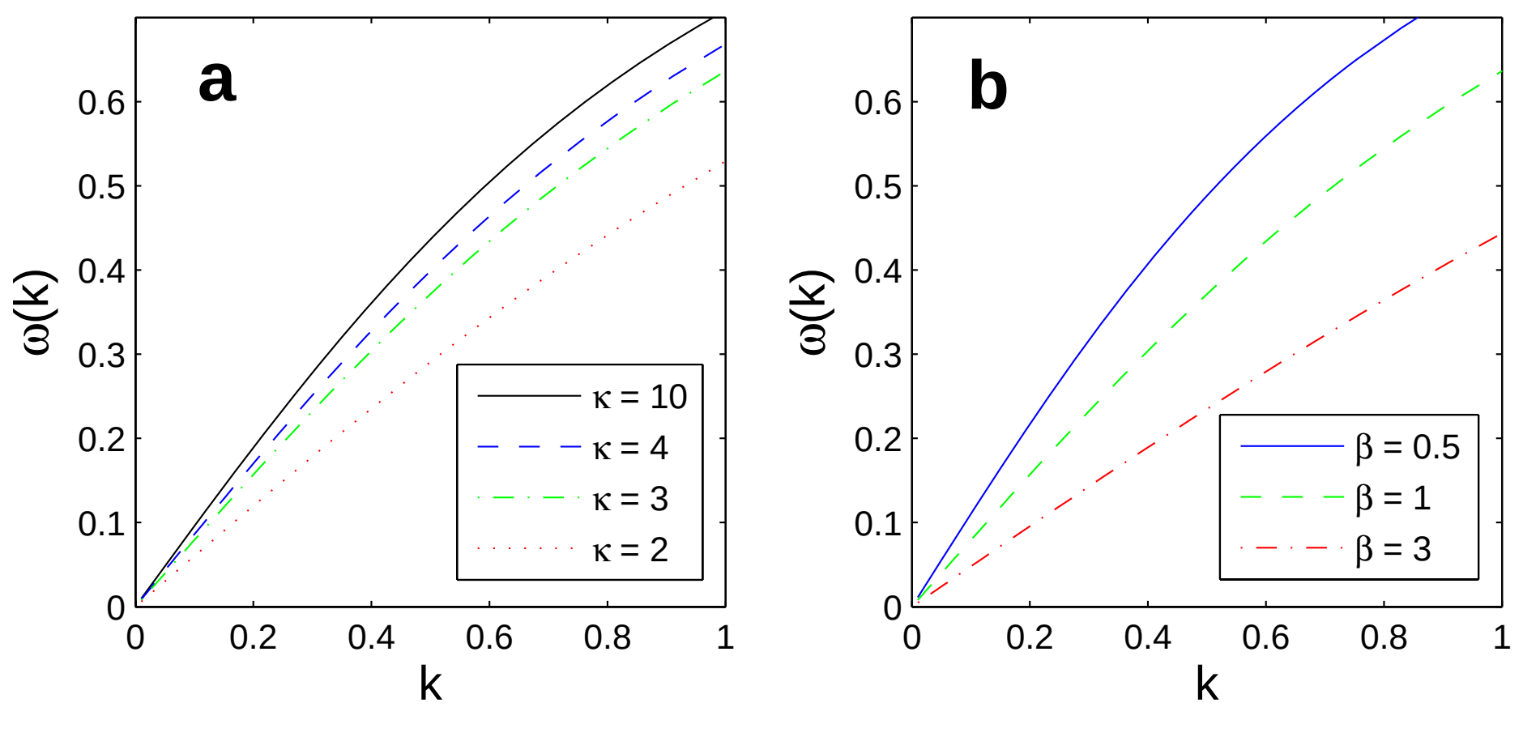

FIG. 1: (left) Variation of the dispersion function curve for different values of κ. Here, $\sigma = 0.01$ and $\beta = 1$. (right) Variation of the dispersion function curve for different values of β. Here, $\sigma = 0.01$ and $\kappa = 3$.

4. Nonlinear Pseudopotential Technique

Considering Eqs. (1)–(4) in a stationary frame ($\xi = x - Mt$) traveling at a constant velocity M. Applying the appropriate boundary conditions, and integrating equations provide:

$$u = M(1 - \frac{1}{n}), \qquad u = M - (M^2 + 2\phi - 3n^2\sigma + 3\sigma)^{1/2}, \tag{8}$$

$$P = n^3. \tag{9}$$

Combining Eqs. (8)–(9), and substituting into the Poisson's equation leads to the equation of motion, and integrating it yields the energy balance equation:

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \Psi(\phi) = 0, \tag{10}$$

where the Sagdeev pseudopotential $\Psi(\phi)$ reads as

$$\begin{aligned}\Psi(\phi) = (1+\beta)\phi + \beta\left[1 - \left(1 + \frac{\phi}{-\kappa + \frac{3}{2}}\right)^{-\kappa+3/2}\right] \\ + \frac{1}{6\sqrt{3\sigma}}\left[\left(M + \sqrt{3\sigma}\right)^3 \pm \left(M - \sqrt{3\sigma}\right)^3 \right. \\ \left. - \left(2\phi + \left[M + \sqrt{3\sigma}\right]^2\right)^{3/2} \mp \left(2\phi + \left[M - \sqrt{3\sigma}\right]^2\right)^{3/2}\right].\end{aligned} \tag{11}$$

5. Soliton Existence

We investigate the conditions for existence of solitons. We need to ensure that the origin at $\phi = 0$ is a local maximum of Ψ in Eq. (11), i.e. $\Psi'(\phi) \equiv d\Psi/d\phi = 0$ and $\Psi''(\phi) \equiv d^2\Psi/d\phi^2 < 0$ at $\phi = 0$ [8, 9, 10]. This imposes the condition

$$F_1(M) = -\left.\Psi''(\phi)\right|_{\phi=0} = \frac{\beta(\kappa - \frac{1}{2})}{\kappa - \frac{3}{2}} - \frac{1}{M^2 - 3\sigma} > 0. \tag{12}$$

Eq. (12) provides the minimum value for the Mach number:

$$M_1 = \left[\frac{\kappa - \frac{3}{2}}{\beta(\kappa - \frac{1}{2})} + 3\sigma\right]^{1/2}. \tag{13}$$

We obtain the largest possible value of M through $F_2(M) = \Psi(\phi)|_{\phi=\phi_{\max}} > 0$. This yields the following equation:

$$\begin{aligned}F_2(M) = -\frac{1}{2}(1+\beta)\left(M - \sqrt{3\sigma}\right)^2 - \frac{4}{3}M^{3/2}(3\sigma)^{1/4} \\ + \beta\left(1 - \left[1 + \frac{\left(M - \sqrt{3\sigma}\right)^2}{2\kappa - 3}\right]^{-\kappa+3/2}\right) + M^2 + \sigma.\end{aligned} \tag{14}$$

Solving Eq. (14) provides the upper limit $M_2(\alpha, \kappa)$ for the Mach number. The existence condition ($M_1 < M < M_2$) is obtained by satisfying $F_1(M) > 0$ and $F_2(M) > 0$.

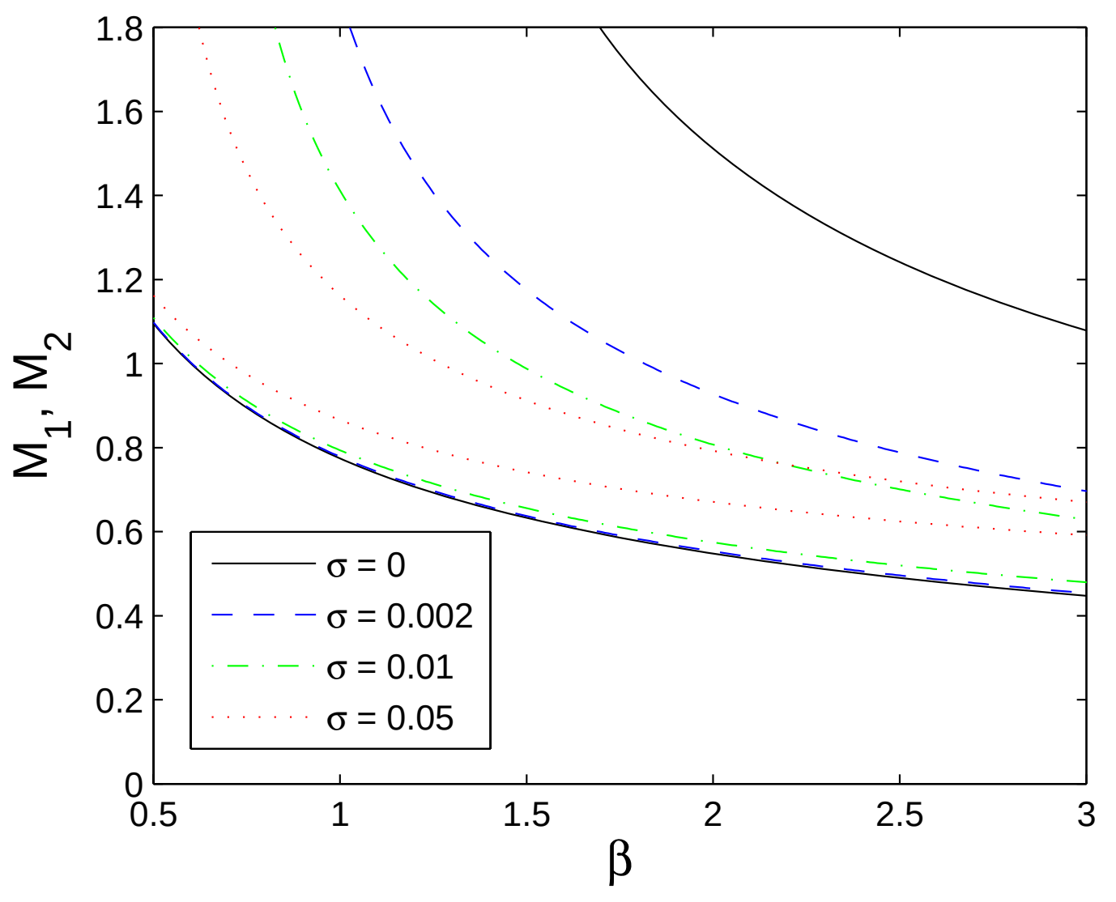

FIG. 2: Variation of the lower limit M_1 (lower curve) and the upper limit M_2 (upper curve) with β for different temperature ratio σ. Here, $\kappa = 3$.

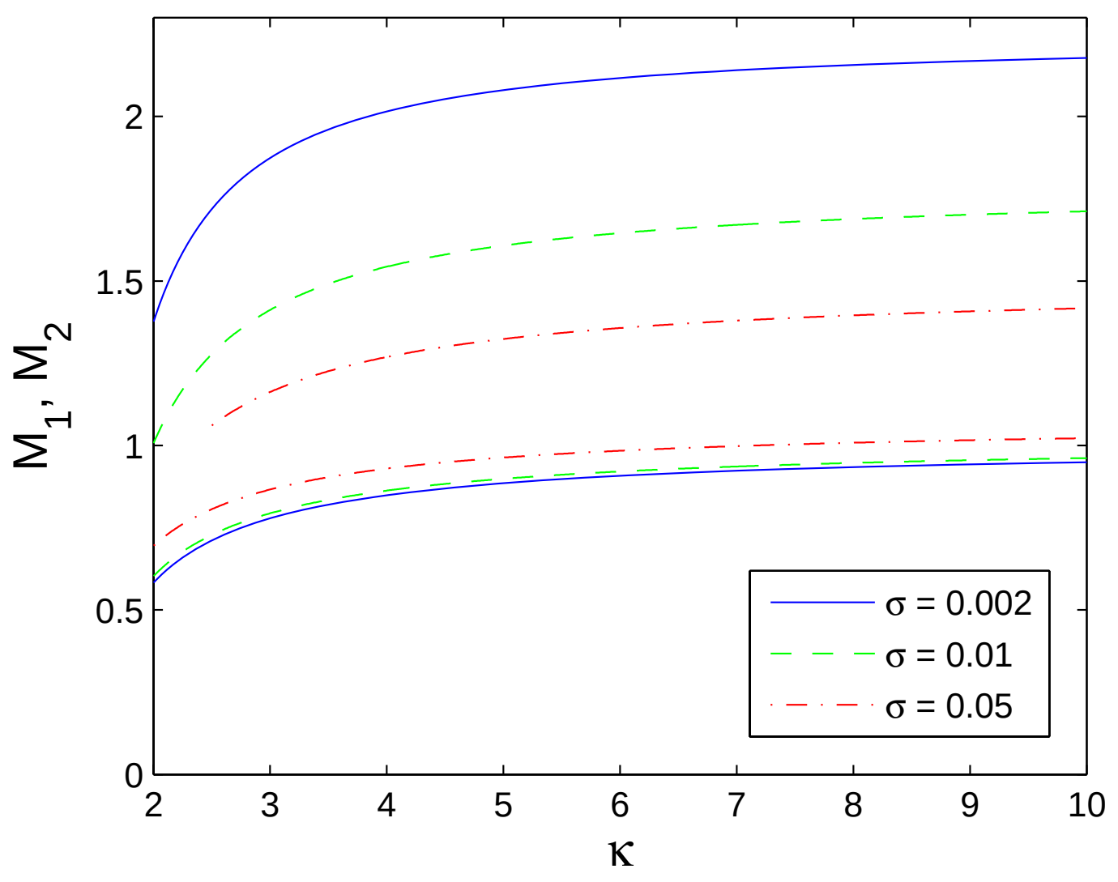

FIG. 3: Variation of the lower limit M_1 (lower curve) and the upper limit M_2 (upper curve) with κ for different temperature ratio σ. Here, $\beta = 1$.

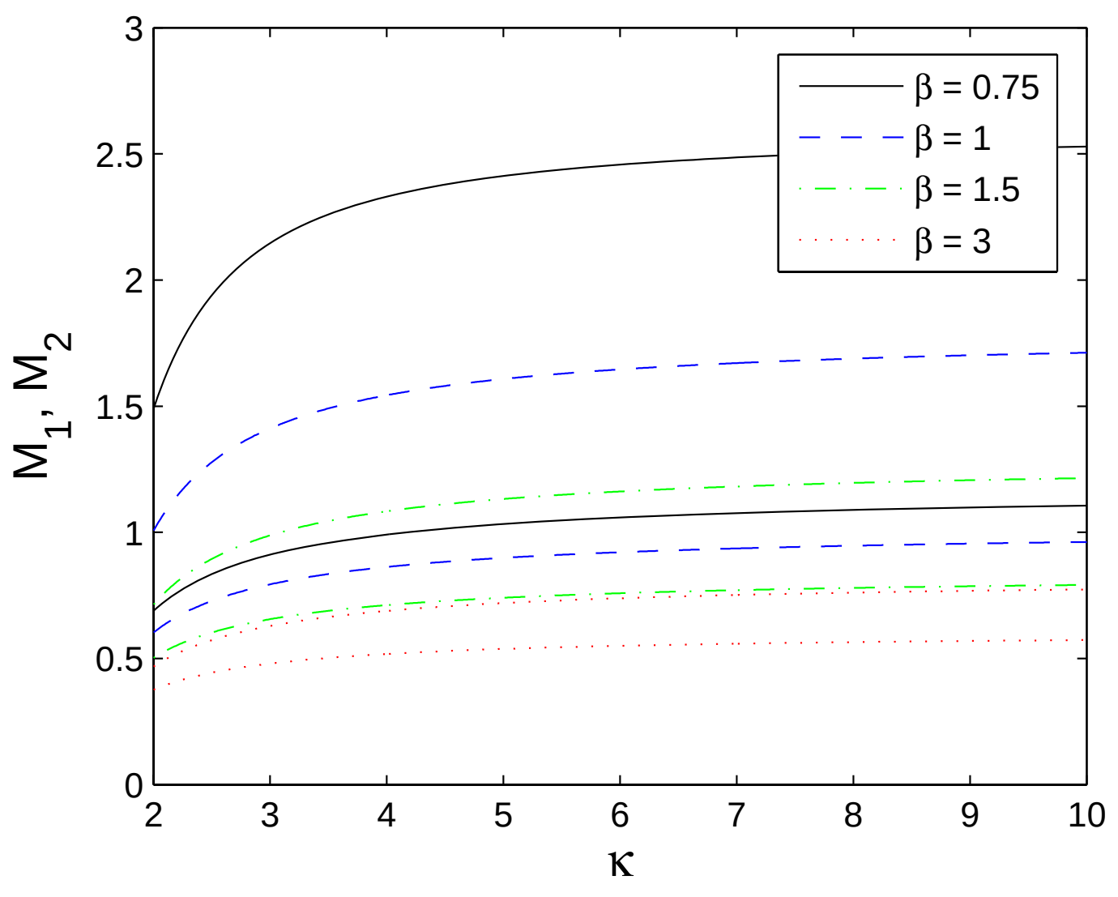

FIG. 4: Variation of the lower limit M_1 (lower curve) and the upper limit M_2 (upper curve) with κ for different density ratio β. Here, $\sigma = 0.01$.

6. Parametric Investigation

6.1 Thermal effect (via σ)

The amplitude of the Sagdeev potential decreases with the increase in σ. Hence, we can see a decrease in the excitations' structure amplitude with increase in σ.

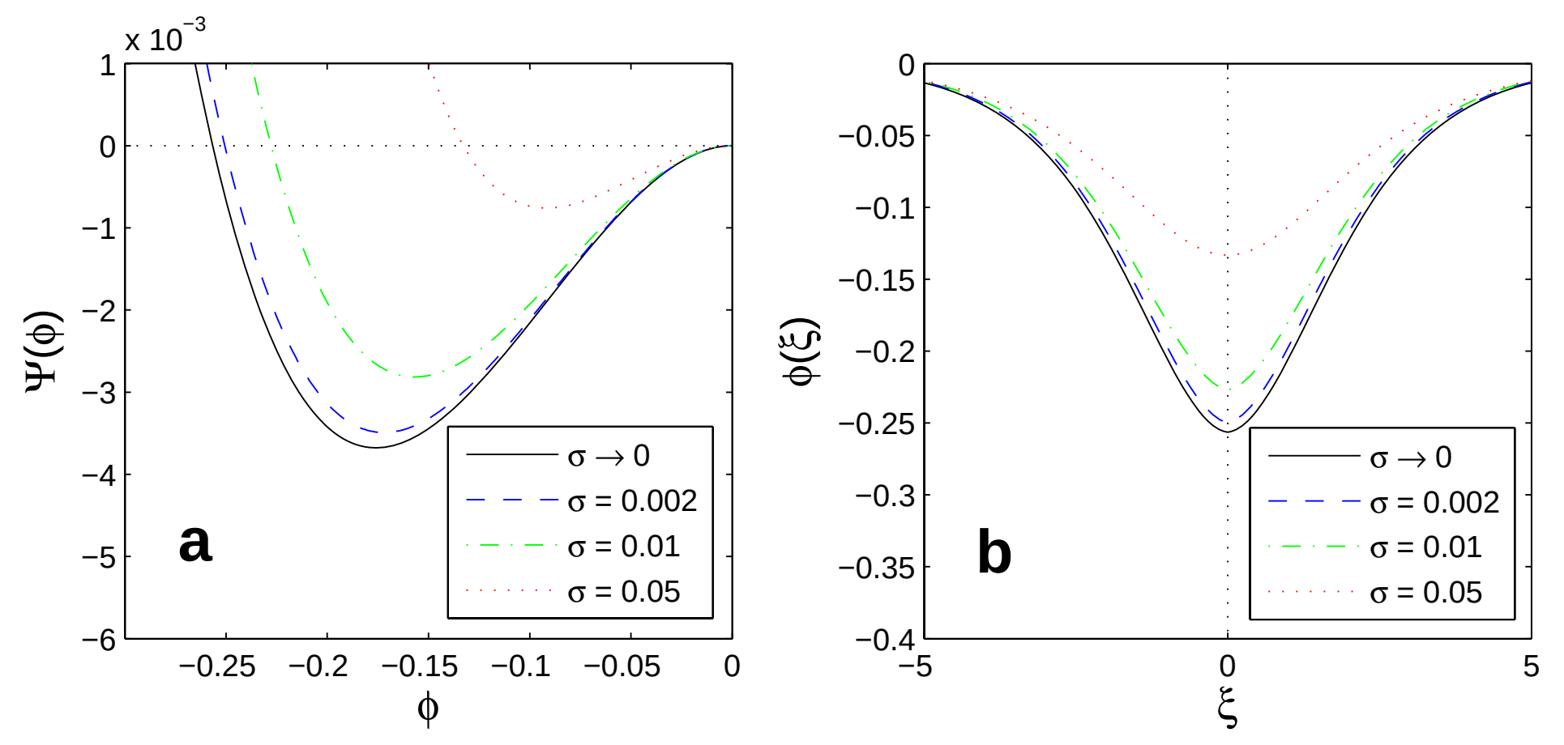

FIG. 5: (left) Variation of pseudopotential $\Psi(\phi)$ with ϕ for different temperature ratio σ. (right) Variation of the electron-acoustic potential ϕ with ξ for different temperature ratio σ. Here, $\beta = 1$, $\kappa = 3$ and $M = 1$.

6.2 Superthermality effect (via κ)

The electrostatic solitary wave amplitude $|\phi_{\mathrm{m}}|$ decreases with weaker superthermality (higher κ).

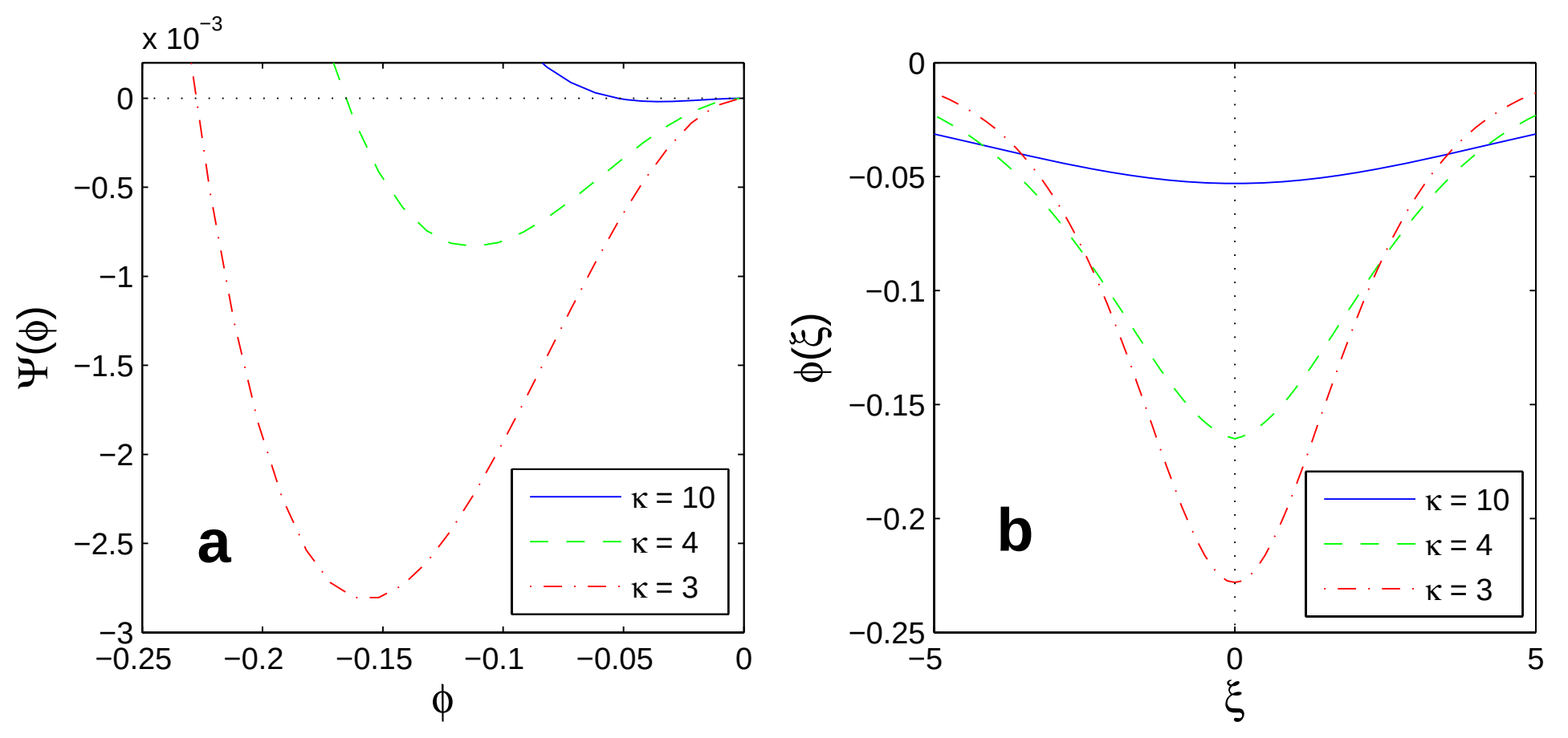

FIG. 6: (left) Variation of pseudopotential $\Psi(\phi)$ with ϕ for different κ. (right) Variation of potential ϕ with ξ for different κ. Here, $\sigma = 0.01$, $\beta = 1$, and $M = 1$.

6.3 Hot electrons percentage effect (via β)

An increase in the number density of the hot electrons leads to a raise in the potential amplitude.

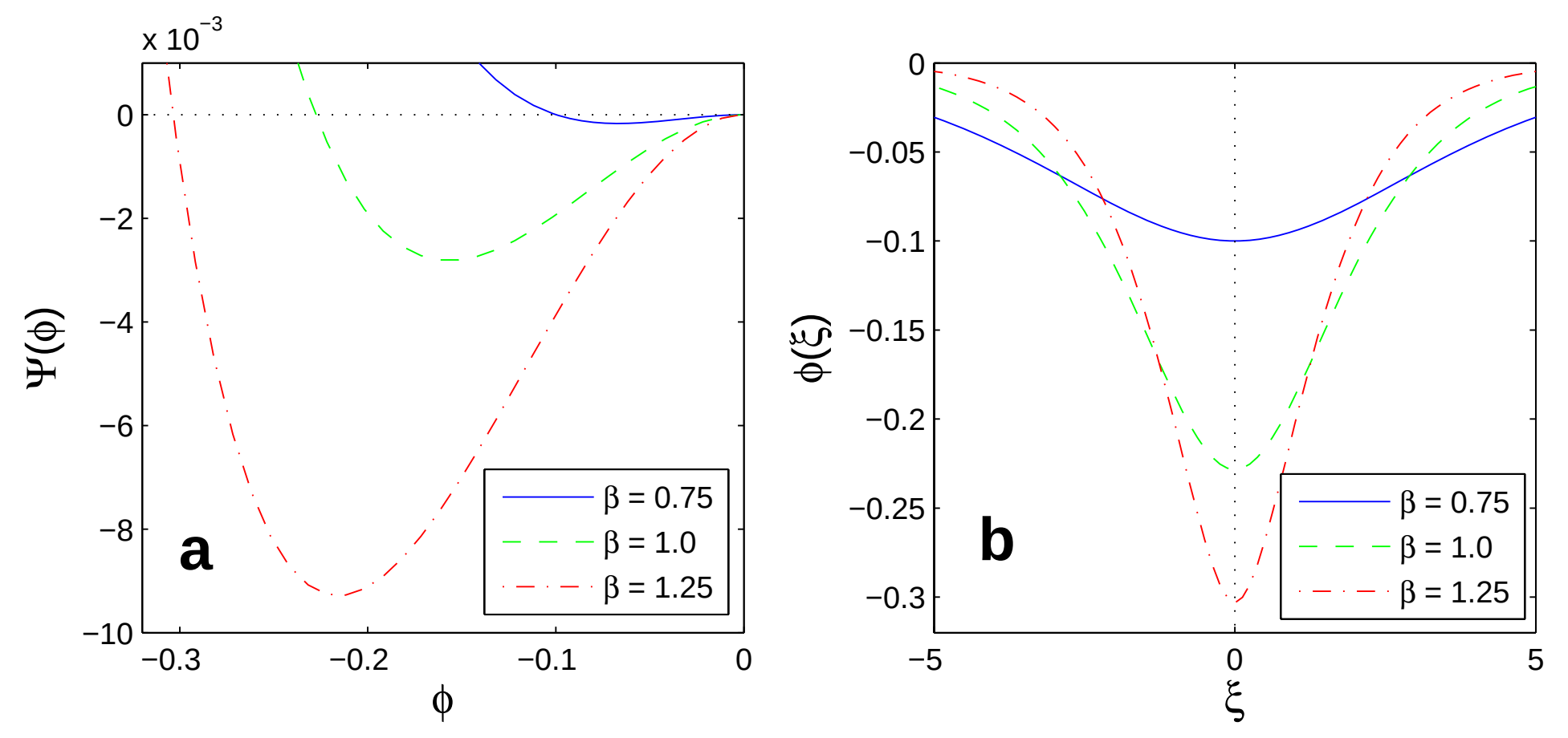

FIG. 7: (left) Variation of pseudopotential $\Psi(\phi)$ with ϕ for different density ratio β. (right) Variation of potential ϕ with ξ for different density ratio β. Here, $\sigma = 0.01$, $\kappa = 3$ and $M = 1$.

7. Conclusions

In the linear analysis, we see that the slope of the dispersion curve shifts downwards with either a decrease in κ or an increase in β. It is observed that the frequency (and the phase speed) of the EAWs decreases due to increase in superthermality and the presence of a strong hot-component of electrons.

A Sagdeev pseudopotential method is used to investigate the existence of large amplitude solitary waves. The existence domain for negative solitons was shown to become narrower with stronger superthermality in the electron distribution, higher number density of hot electrons, and with increase in the temperature ratio σ.

The dependence of solitary wave structures on the plasma parameters is investigated. The potential amplitude increases with increase in hot electron temperature (decreasing σ), superthermality index (decreasing κ), and hot-component of electrons (increasing β).

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